Is Current the Conduction of Charge?

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Abstract

Most scientists believe that current flow is the conduction of charge. Mathematics and physics imply otherwise. The mathematics of magnetism requires another component, the displacement current that depends on the time derivative of the electric field. Displacement current is an experimental reality. Displacement current is needed for light to propagate in a vacuum devoid of charges. Maxwell himself defined true current to include displacement current. Mathematics and Maxwell both show that displacement current is an essential part of electrodynamics

Displacement current is evident in the transients seen in experiments on time scales of RC and faster where R is resistance and C is typically 10 pF. R is often 100 kohms, RC is then 10^{-6} seconds. Computers function on much faster time scales than that, 10^{-9} seconds and faster, where displacement currents certainly cannot be ignored. Practical circuit models today provide that displacement current by changing the definition of resistors. The idealized circuits of topological models change the definition of resistors by supplementing them with stray (or parasitic) capacitances that are used to provide the transients so prominent in experiments. The supplementary elements are subjective, ill-defined, and incomplete, although useful. They do not provide a proper (mathematically defined) basis for the circuit theory that makes possible the billions of integrated circuits of computers and smart phones.

The precise mathematical definition of current is Maxwell's true current. This definition of current avoids the practical and mathematical difficulties of poorly defined supplementary components. It is needed to understand current flow in biological systems and chemical reactions. It is likely to improve the design and speed of the circuits of our computers.

Most scientists believe that current flow is the conduction of charge. Electrodynamics does not support this view. The equation defining magnetic forces implies an additional component of current, the displacement current. Displacement current is present in a vacuum devoid of charge. It depends on the time derivative of the electric field and allows electromagnetic waves to propagate through a vacuum where there are no charges to conduct current. Maxwell emphasized the importance of displacement current when he defined true current. He could hardly have used a more vivid adjective than "true". He wrote " … the true electric current, that on which the electromagnetic phenomena depend, is not the same thing as the current of conduction" Vol. 2, Section 610, p. 232 of [1]; also see section 783 p, 385.

Magnetism $\mathbf{B}$ is defined by the field equations of electrodynamics. Magnetism does not have any sources in the usual meaning of the term.

$$\mathbf{div\,B} = 0 \tag{1}$$

Magnetism is driven by its curl, not by sources of its divergence.

$$\frac{1}{\mu_0}\mathbf{curl\,B} = \frac{1}{\mu_0}\,\text{'}\boldsymbol{true\ current}\text{'} = \overbrace{\mathbf{J}}^{\substack{Conduction\\ Current}} + \overbrace{\underbrace{\varepsilon_0(\varepsilon_r - 1)\frac{\partial\mathbf{E}}{\partial t}}_{\substack{Material\\ Approximate}} + \underbrace{\varepsilon_0\frac{\partial\mathbf{E}}{\partial t}}_{\substack{Universal\\ Exact}}}^{Displacement\ Current} \tag{2}$$

The obvious definition of current is the driving force for the curl, namely the entire right-hand side of the curl eq. (2).

The displacement current itself involves two components with very different properties. It includes a universal component $\varepsilon_0\partial\mathbf{E}/\partial t$, as exact and universal as the Maxwell equations themselves. This component exists in a vacuum as well as everywhere else and allows signals to propagate as light waves in regions that contain no charge whatsoever.

The displacement current also includes an approximate component $\varepsilon_0(\varepsilon_r - 1)\,\partial\mathbf{E}/\partial t$, best viewed as a placeholder for the complex dielectric properties of real materials [2-

17]. The dielectric properties include optical spectra that vary dramatically with wavelength (frequency), as described in [9] Section L.2.4.A-E., p. 241-275. They are as complex as the spectra of organic chemicals [2, 8, 11-14, 16], and as diverse. They obviously cannot be defined by a single real dielectric constant.

The time-dependent displacement currents are large and cannot be neglected in the circuits of our technology [18-33]. Experiments with resistances R show a substantial time dependence at time constants $=$ RC where C is about 10 pF. R is often 10^5ohms, making RC one microsecond. Our computers function on a much shorter time scale than one microsecond, often 0.1 nsec. Displacement currents cannot be ignored in our computer circuits. Mathematics, physics, engineering, and experiments show that the displacement currents in eq.(2) $\varepsilon_r \varepsilon_0\, \partial \mathbf{E} / \partial t$ are as important just as Maxwell said they were.

Most scientists have overlooked Maxwell's definition of true current. This is not surprising: scientists are people. People are often reluctant to let go of ideas, particularly those memorized before they are trained as scientists. Studying electricity in school, students are taught that current can be understood as the flow of water. But current flow is not like water flow on the time scale RC of our computers. The image of water flow image leaves out the coupling produced by displacement current. As Horowitz and Hill [34] put p. 579-589: "Signals within an instrument can get around handsomely by electrostatic coupling." The proper definition of current then should include the displacement currents that produce coupling so evident in real instruments.

Using the image of water flow, scientists account for time dependence seen in networks of resistances by changing their definition of resistors. That definition replaces the idealized resistors in the simplified topological networks of elementary circuit analysis. The revised definition of resistors provides the displacement current observed experimentally in transients. The revised definition of resistors supplements the resistor with parasitic (often called stray) capacitors that produce the transients seen in experiments. The names 'parasitic' and 'stray' cast a shadow on the significance of displacement currents, while the equation of magnetism eq.(2), casts a spotlight on the importance of the time derivative as a chief characteristic of electrodynamics, as Maxwell said [1], *loc. cit*.

Science seeks to be objective, well defined, and complete. The supplementary elements are subjective, ill-defined, and incomplete, quite undesirable features for the foundation of a technology as vast as electronics. Circuit theory deserves better and it needs better. A mathematically precise definition of current will support the design of more rapid circuits that do what they are supposed to.

The 'parasitic' capacitances also hide the generality and physical significance of displacement current that Maxwell discovered and believed so important. They lump together the universal unavoidable displacement current given exactly by $\varepsilon_0 \partial \mathbf{E}/\partial t$ of eq. (2) with the remarkably diverse material properties of physical resistors crudely approximated by $\varepsilon_0(\varepsilon_r - 1)\,\partial \mathbf{E}/\partial t$. If one thinks only of material displacement current, it is easy to accept a crude approximation that ignores the universal exact displacement current given by $\varepsilon_0 \partial \mathbf{E}/\partial t$.

Mathematics should be used to study true currents because they have a special property. True currents are solenoidal divergence-free fields. This mathematics played an important role in Maxwell's thinking, in my view. Maxwell was eager to incorporate Faraday's visualization of lines of force in a well-defined mathematical theory and hoped that the properties of solenoidal divergence-free fields would do that. (We now know that hope cannot be completely fulfilled in three dimensions [35, 36]. Two dimensions are a different matter.) The lines of force of solenoidal fields become mathematically precise when the divergence operator is applied to the curl equation. The divergence of the curl is always zero—easily verified by direct substitution in the Cartesian definition of the vector functions. The div curl zero identity is studied at many levels of abstraction in the mathematics literature [36-38].

Using mathematics, we can see that true current is a solenoidal divergence-free field under all conditions and at all times that the Maxwell equations are true.

$$\mathbf{div}\ \mathbf{curl}\ \mathbf{B} = 0 = \mathbf{div}\ \mu_0 \left(\mathbf{J} + \varepsilon_r \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right) = \mathbf{div}\ \mathbf{J}_{\boldsymbol{true}} \tag{3}$$

$$\mathbf{div}\ \ \mathbf{J}_{\boldsymbol{true}} = 0 \tag{4}$$

Mathematicians established the important role of divergence-free fields in the mathematics of partial differential equations by studying incompressible fluids in fluid mechanics and hydrodynamics [36, 38-46]. Fluids do not always flow as simply as they do in incompressible fluids. Fluids do not always flow in generalized circles or circuits. The properties of incompressible divergence-free fluids allow analysis that is impossible for fluids in general. It is natural that mathematicians have worked so hard to exploit the special properties of solenoidal fields.

The simplification allowed by the analysis of divergence-free incompressible systems is so helpful that it is easy to overlook the approximations involved. In fact, real fluids are not incompressible at all times. Real fluids compress rapidly before they become incompressible and divergence-free, typically on a microsecond time scale. The classical simplifications of analysis of incompressible fluids only apply on long time scales.

Maxwell's true current is different. Its properties are true at all times that the Maxwell equations themselves are true. Maxwell's true current forms a flow field that is incompressible and divergence-free ***on all time scales***. True current never accumulates. True current flows in generalized circles—circuits—that end where they begin in a wide variety of systems.

The special properties of incompressible divergence-free flow are particularly striking in a simple one-dimensional series system***. In a series system, true current is the same everywhere,*** even though the physical origins of the currents are entirely different in different components of the series system. Fig. 2 of [47] shows currents in series combinations of wires, resistors, capacitors (vacuum and real), semiconductors, and ionic solutions. The global nature of electrodynamics forces the currents to be equal in every component in a series circuit. The special properties of divergence-free solenoidal fields describe true current in all these components, under all conditions, at all times, no matter how brief, including on the time scale of Brownian motion and chemical reactions [48].

Using circuit approximations that depend on ill-defined parasitic capacitances hides this reality of flow without accumulation, which is practically important as our computer circuits become faster and faster. It also has important effects in chemistry where displacement current is customarily neglected altogether. And it has important effects in biology where displacement current is needed to understand the coordination of contraction in the heart, and skeletal muscle, and the signals of the nervous system.

The breadth and importance of these facts raise the question why so little attention has been paid to the universal properties of the divergence-free flow of true current. Unlike the flows of actual fluids in hydrodynamics, true current is one flow that is always incompressible and ***free of divergence on any time scale*** (in which the Maxwell equations are valid). It is one flow in which the power of solenoidal divergence-free analysis can be applied without approximation or hesitation to draw general conclusions.

It is natural to wonder about the physical nature of total current. That question occupied an important place in physics as the theory of special relativity emerged from the Maxwell equations [49-51]. That nature is still not clear, at least to me. Fortunately, the nature of the current carrier is unimportant in most applications and circuits. Applications depend on the current flow, whatever the current carrier, as long as eq.(2) applies. The definition of an (a)ether [49-51] is not needed to understand the properties of true current in circuits.

No matter what is flowing, the general properties of incompressible true current are important in the practical engineering of circuits in two dimensions. Divergence-free fields have particularly simple properties in two-dimensional systems of finite size [35, 36]. In such systems flows end where they begin, moving in circuits that are generalized circles. The incompressible solenoidal properties of true current justify the circuit theory of idealized topological networks that makes up so much of electrical engineering. The divergence-free properties of true current link the Maxwell equations with the usual circuit theory of electrical engineering taught to many millions of students in the last hundred years [18-33].

Circuit theory is usually taught using idealized topological circuit diagrams [25, 28, 34]. “Black magic” [23] is then needed to convert these idealized topological circuit diagrams into practical circuits that work [24, 30, 31, 33, 52, 53]. “X-chapters” [52] and “black magic” have become a crucial part of “the art of electronics” [34, 52] now that most of our circuits function in nanoseconds. Maxwell’s definition of current can make that black magic transparent. Using his mathematically precise definition of true current is likely to help speed up the design and function of practical circuits.

The general properties of incompressible current also have an integral role in biology. In long cells, like nerve and skeletal muscle, the balance of total current is responsible for the propagation of the nerve signal, the initiation of muscle contraction and the coordinated contraction of the heart that allows it to pump blood. The balance of total current is the essential physics of the Hodgkin Huxley treatment of the action potential. Because true current is incompressible, all the true current that enters a mitochondrion [54] must leave [55, 56]. Flows of individual charges can accumulate for a while, to be balanced out at a later time. The total current cannot accumulate. At any microsecond, it must balance.

If an inward current increases, an outward current must balance it 'instantaneously' on any time scale at which the Maxwell equations apply. The Maxwell equations couple currents independently of the physical nature of conduction current. Indeed, Maxwell coupling is responsible for most of the coupling of ion movement seen in bulk solution. Any simulation of molecular dynamics shows enormous fluctuations in electric potential (think 1 volt in 1 picosecond) that move ions much more than diffusion can. Diffusive potentials are roughly 25 millivolts. These electrical potentials are not included in classical [57-59] or modern theories of multicomponent diffusion, because they are too complex to analyze charge by charge as described in [60]. The Maxwell coupling that makes it possible to analyze the electrical forces correctly depends on the properties of the equations of magnetism eq. (2) & (4). The coupling cannot be analyzed charge by charge. There are too many charges and far far too many interactions of charges to deal with.

Because of the enormous electric fields, Maxwell coupling is far more important than diffusive collisions in coupling the movement of different ions, and different types of ions. Using the coupling implied by the Maxwell equations (2) & (4), the electrical forces are much easier to specify and analyze than the diffusive forces. Indeed, diffusive, and mechanical coupling are often named, but not analyzed. Vaguely defined mechanical or chemical ('allosteric') interactions that are invoked in the protein literature can hide underlying Maxwell coupling that is specific and easy to analyze with Kirchhoff's current law and its generalization, eq. (4).

As usually performed in biological applications, molecular dynamics based on Coulomb's law do not reveal Maxwell coupling because Coulomb's law is "to be used only for statics", as Feynman [61] vividly describes over three pages in Section 15-6. Indeed, most protein molecular dynamics do not actually use Coulomb's law. Molecular dynamics usually assumes periodic systems but Coulomb's law has no periodic solutions!

The particle-in-cell methods of plasma physics have been used to deal with the full Maxwell equations for a long time and are now available in computer programs and tutorial format suited for new applications [62-64]. Particle-in-cell methods need to be adapted [65] to the molecular dynamics of ions and proteins before Maxwell coupling can be studied in atomic detail.

Maxwell coupling is just as crucial in mitochondria [54] as it is in engineering circuits and molecular dynamics. Mitochondria generate ATP, the nearly universal currency of chemical energy in biology. The total flow of current, i.e., true current, into a mitochondria is what provides the outward flow of current and power that drives the chemical reaction that synthesizes ATP in ATPsynthase. The coupling of the many components of the electron transport chain [54] to ATPsynthase is electrical, not chemical, not mechanical, not allosteric. Similar coupled flows are likely to drive other systems like the chloroplasts of plants and the flagellar motor in bacteria and cells. By using Maxwell's true current, scientists could understand these systems quantitatively, where only qualitative description is available today.

It seems about time that scientists should adopt a definition of current consistent with the properties of magnetism and electricity. The correct definition of current will make the circuits of our computers faster and easier to design. It will allow better understanding of many biological phenomena now vaguely called allosteric. It will change our understanding of chemical reactions and changes in chemical bonds, since they are rapid and involve large displacement currents too large to ignore.

Adopting the classical yet new idea of Maxwell's true current could help physics, chemistry, engineering, and biology. While people are reluctant to let go of what they have been taught, scientists love to experiment. True current is a place to start.

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